XI. *On the action of crystallized surfaces upon light. By* David Brewster, *LL. D. F. R. S. Lond. and Edin. In a letter addressed to the Right Hon. Sir* Joseph Banks, *Bart. G. C. B. P. R. S. &c. &c. &c.*

Read February 25, 1819.

My dear Sir,

It has been remarked by Malus, in his Theory of Double Refraction, " that the action which the first surface of Iceland " spar exercises upon light, is independent of the position of its " principal section ;—that its reflecting power extends beyond " the limits of the polarising forces of the crystal, and that as " light is only polarised by penetrating the surface, the forces " which produce extraordinary refraction begin to act only at " this limit." He also observes, that " the angle of incidence " at which Iceland spar polarises light by partial reflection, is " 56° 30′; that it then comports itself like a common transpa- " rent body; and that whatever be the angle comprehended " between the plane of incidence and the principal section of " the crystal, the ray reflected by the first surface is always " polarised in the same manner." *

These conclusions, obtained experimentally by an author of such distinguished eminence, I should naturally have received as established truths, had I not been led, by a series of experiments made before the perusal of his work, to opinions of an opposite kind. My experiments indicated an extension of the polarising forces *beyond* the crystal; and I

* *Theorie de la Double Refraction*, pp. 240, 241.

was thus induced to question the accuracy of Malus's views, and to repeat the experiments upon which he had founded them. The results of this investigation, while they have overturned the opinions hitherto adopted, have at the same time led to the establishment of several points both of theoretical and practical importance.

In giving an account of these results, I shall *first* consider the effects produced upon transmitted light by a change in the mechanical condition of the surfaces of crystals, and then establish the laws according to which the interior forces affect and modify the forces which produce partial reflection.

Sect. I. *On the effects produced upon transmitted light, by a change in the mechanical condition of the surfaces of crystals.*

If we take a hexaedral prism of *nitrate of potash*, and observe a luminous object through two of its inclined surfaces that have a good natural or artificial polish, we shall perceive two distinct and perfectly formed images. If we now roughen these two surfaces, and cement upon each of them a plate of glass by means of *balsam of capivi*, the character of the two images will be greatly changed. The image that has suffered the greatest refraction will be as distinct as before, but the other image will be either of a faint reddish colour, or wholly invisible, according to the degree of roughness induced upon the refracting surfaces. When *oil of cassia* is used instead of the balsam, the least refracted image, if it was visible before, will now be completely extinguished.

By substituting pure *alcohol*, or the *white of an egg*, instead of the balsam, the least refracted image will become distinct, and the most refracted image will be either a mass of nebulous light, or almost invisible. A result nearly similar will be obtained with water, notwithstanding its effect in dissolving the little prominences which constitute the superficial roughness of the crystal.

In order to explain these phenomena, we must recollect that the index of refraction for the ordinary image of nitre is 1.511, and that of the extraordinary image 1.328. When the rough surface of the nitre is covered with balsam of capivi, which has nearly the same index of refraction as the ordinary image, the same effect is produced as if the rough surface had been polished for the ordinary rays. All the little pits or depressions in the rough surface being filled up with balsam, the ordinary rays suffer little or no refraction in penetrating the crystal, and therefore the image which they form will be as clear and distinct as in the first experiment. But since the index of refraction for the extraordinary image is much less than that of the balsam, the rays of which it is composed will not enter the crystal undisturbed, but will be scattered in the same manner as if its surface was rough, and had a refractive power corresponding to the difference between the index of refraction for the extraordinary ray, and the index of refraction for the balsam. When water or alcohol is substituted in room of the balsam, the effects now described are interchanged, the roughness being removed for the extraordinary rays by the application of a fluid of the same refractive density, while the rays that form the ordinary image are dispersed by the refractions which still exist at the rough surface of the crystal.

These effects will be still better understood by supposing the crystal to consist of an extraordinary and an ordinary medium, arranged in alternate strata. When the superficial polish of both these media is removed, the application of the *balsam* restores, as it were, the polish of the ordinary medium, without restoring that of the extraordinary medium; while the application of the *alcohol* restores the polish of the extraordinary medium without restoring that of the ordinary medium.

When the refractive power of the fluid is intermediate between that of the two media, the ordinary and the extraordinary image will be equally indistinct; and we have it in our power to alter the distinctness of either of the images, by varying the refractive force of the interposed fluid.

If the plane of incidence is not perpendicular to the axis, a variation in the angle of incidence will produce a variation in the index of refraction of the extraordinary ray; and, since the refractive power of the interposed fluid suffers no change, the extraordinary image must become more or less distinct, according as its index of refraction is made to approach more or less to that of the fluid, by varying the inclination of the refracted ray to the axis.

From the preceding experiments, which have been repeated with the same results with *calcareous spar*, *arragonite*, and many other crystals, we may deduce the following conclusions:

1. The force of double refraction and polarisation extends not only without the interior limit of the ordinary refractive force, but also without the surface of the crystal.

2. The force of double refraction and polarisation emanates from the surface of bodies, though its intensity depends upon the inclination of the surface to the axis of the crystal.

3. The ordinary or the extraordinary image may be ex-

tinguished at pleasure in any doubly refracting crystal; and the crystal is thus converted into a singly refracting crystal, like certain specimens of agate.

4. In soft crystals that do not admit of a perfect polish, the distinctness of any of the two images may be made a maximum, by giving the crystal the best polish of which it is susceptible, and then cementing plates of glass upon its surfaces, by a transparent cement of the same refractive power as that of the pencil which is to be rendered most distinct. If it is required to make the two images equally distinct, the refractive power of the cement must be a mean between that of the ordinary refraction, and the extraordinary refraction which corresponds to the angle which the refracted ray forms with the axis of double refraction.

5. All doubly refracting crystals consist of an ordinary and an extraordinary medium, alternating with each other, and varying in density according to a law which I have described in another paper.*

I consider the optical structure of *agate* as demonstrating the existence of two media. In quartz, the two media are equally perfect and transparent; but in certain specimens of agate, the one medium is seen in a separate state from the other, and broken down into small portions like the figures 333. The light which passes through these portions, is evidently acted upon by a different refractive power from that which

* The paper here alluded to, was laid before the Royal Society of Edinburgh on the 16th of March, 1818; but as it could not have been understood without the preceding experiments, its publication was necessarily delayed. The theory which it contains embraces also the complex phenomena which arise from the combination of two or more axes. See the *Phil. Trans. Lond.* 1818, p. 264.

passes through the rest of the crystal, and is polarised in a transverse plain.*

SECT. II. *On the influence of the polarising force of doubly refracting crystals, upon the polarising force which accompanies partial reflection.*

The experiments in the preceding section could not fail to throw a doubt upon the identity of action exercised upon reflected light by crystallized and uncrystallized surfaces. I was therefore led to a more minute investigation of the subject, and obtained a series of very unexpected results, which I shall explain under the three heads into which they naturally arrange themselves.

1. *On the change produced upon the polarising angle by the interior forces of doubly refracting crystals.*

In order to examine with care the superficial action of calcareous spar, I exposed several surfaces by cleavage, and having selected the one that had the most perfect polish, I covered all the other sides of the rhomb with black wax, and measured the polarising angles in planes variously inclined to the principal section. The following are the results of a great number of observations:

Position of the crystal.	Azimuth.	Polarising angle.	No. of observations.
Short diagonal in plane of reflection	0°	57° 14′	39
One of the edges in plane of reflection	50 57½′	58 32	15
Long diagonal in plane of reflection	90	59 32	37
Difference between the greatest and least polarising angle - - - -		2° 18′	

* See *Phil. Trans.* 1813, p. 104; 1814, p. 191; and *Edin. Trans.* vol. vii. p. 298, 9.

The following observations were made with rhombs taken from a different mass of calcareous spar:

Position of the crystal.	Azimuth.	Polarising angle.	No. of observations.
Short diagonal in plane of reflection	0°	57° 36′	5
One of the edges in plane of reflection	50 57½′	58 50	7
Long diagonal in plane of reflection	90	59 44	7

In these experiments the results were the same, whether the obtuse angle of the rhomb was nearest or farthest from the eye, or whether it was to the right or left hand of the observer.

In order to determine the angle of polarisation for surfaces differently inclined to the axis, I selected some fine Faroë crystals of calcareous spar from the cabinets of Sir George Mackenzie and Mr. Allan. One of these crystals was an acute dodecahedron, having a highly polished surface inclined about 5° to the axis, and with it I obtained the following results.

Position of the crystal.	Polarising angle.	No. of observations.
Axis in plane of reflection -	54° 18′	13
Axis perpendicular to plane of reflection - - -	58 14	10

I was now desirous to obtain a surface perpendicular to the axis; but I have searched in vain for such a specimen, and have invariably found that the summit of the prism is rough and unpolished. With a surface polished by art, and cut perpendicular to the axis, I found the polarising angle about 58° 15′; but I do not regard this result as deserving any particular attention.

In order to supply the defect of natural faces, I ground and polished a great variety of surfaces, inclined at all angles to the axis; but the results clearly proved that the peculiar action of the surfaces, in varying the polarising angle, is exhibited only by the highly polished faces which are sometimes obtained from cleavage, or which occur in perfect crystals.

The following observations were made with a fine crystal of *Chromate of lead:*

Position of the crystal.	Polarising angle.	No. of observations.
Axis of prism in plane of reflection	67° 48′	4
Axis of prism perpendicular to the plane of reflection -	65 42	4

In the first of these positions a great quantity of brilliant blue light remained unpolarised, whereas in the second position the whole of the pencil suffered complete polarisation.

2. *On the change produced upon the polarisation of the reflected ray, by the interior forces of doubly refracting crystals.*

Since the extraordinary force in calcareous spar was thus shown to extend to such a distance beyond the surface as to modify the polarising angle produced by superficial reflection, it became extremely probable that the polarisation of the reflected ray might suffer some change from the same cause: but after the most careful observation, I could not discover the slightest indication of such an effect. Upon reflecting farther, however, on the nature of the change which I had expected, it occurred to me that the action of the ordinary reflecting force was so powerful, as to mask the influence of the inferior force which emanated from the axis, and that the effect of the one might be rendered visible by diminishing the intensity of the other. I accordingly introduced a film of oil of Cassia between a glass prism and the surface of the spar, and having inclined the prism at a very small angle to that surface, I thus separated the image formed at the common surface of the prism, and the oil from the image formed at the common surface of the oil and the spar. The effect was exactly what I had anticipated. The influence of the ordinary reflecting force was reduced almost to nothing, and the light reflected from the separating surface of the oil and the spar, was polarised at an angle of about $45\frac{1}{2}°$, and was almost entirely under the dominion of the force which emanated from the axis. The following were the results obtained with an ordinary surface, inclined 45° $23\frac{1}{2}'$ to the axis.

1. *Azimuth* 0°. When the plane of the principal section is in the plane of reflection, the light reflected at the surface of the oil and the spar is polarised in the plane of reflection, the obtuse solid angle being farthest from the eye. The light of the image is of a faint red colour, and has very little intensity.

2. *Azimuth* 12°. The obtuse angle being farthest from the eye, the reflected pencil is polarised about 45° out of the plane of reflection.

3. *Azimuth* 42°. The reflected pencil is polarised transverse to the plane of reflection, or 90° out of it. The light is now of a yellowish white tint, and is much more intense than in azimuth 0°.

4. *Azimuth* 90°. When the plane of reflection is perpendicular to the plane of the principal section, the obtuse solid angle being either to the right or left hand, the reflected pencil is polarised a little more than 135°, or —45° out of the plane of reflection. The intensity of the pencil is now intermediate between that of azimuth 0° and 45°.

5. *Azimuth* 180°. The obtuse angle being now next the

eye, the pencil is polarised 180° out of the plane of reflection, or it has again returned into that plane.

In passing through the last 45° of azimuth, the polarisation varies very slowly, the change being only about 10°; whereas in passing through the first 42° of azimuth, the polarisation varies no less than 90°, indicating in the most unequivocal manner, as we shall afterwards see, that this change depends upon the angle which the incident ray forms with the axis of the crystal.

The light reflected from the separating surface of the oil and the spar is a *maximum*, when the plane of the principal section is perpendicular to the plane of reflection, and its colour is then nearly white. When these two planes coincide, the intensity of the light is a *minimum*, and its colour is then a faint red; and in intermediate positions, the reflected pencil has both its intensity and its colour of an intermediate character. In the azimuth of 42°, the reflected pencil exhibits a very curious phenomenon when analysed with calcareous spar. Its colour is then yellowish white, and all the *yellow* light is polarised transversely to the plane of reflection. One of the images, however, instead of vanishing, consists of *blue* and *red* light, the *red* vanishing, and the *blue* becoming more brilliant as the analysing prism is turned to the *left;* and the *blue* vanishing, and the *red* becoming more brilliant as the prism is turned to the *right*. This effect arises from the difference in the angles at which the red and blue rays are incident upon the separating surface of the oil and the spar. Each set of rays, therefore, as will afterwards appear, suffers a different change of polarisation, the one being polarised about 87° out of the plane of reflection, and the other 93°.

I have repeated the preceding experiments by substituting in place of oil of cassia, *water, alcohol, castor oil, balsam of capivi*, and *oil of anise seeds*, a series of fluids whose refractive powers increase progressively. With *water*, the light refuses to be polarised completely in the direction of the long diagonal, while it suffers complete polarisation in the direction of the short diagonal. With *alcohol*, the direction of the polarisation is not altered. With *castor oil*, the intensity of the light is greater in the direction of the long diagonal, than in that of the short one; and in the former case, the pencil is polarised at a much greater angle than in the latter. With *balsam of capivi*, in the azimuth of 45°, the pencil is polarised about 15° out of the plane of reflection. In the azimuth of 90°, the pencil is not completely polarised at any angle, but is nearly so in the plane of reflection, and at a considerable angle of incidence. In 0° of azimuth, the pencil is completely polarised in the plane of reflection. With *oil of anise seeds*, in azimuth 45°, the pencil is polarised about 45° out of the plane of reflection. In azimuth 90°, the pencil refuses to be polarised at any angle, and in 0° of azimuth, the polarisation is complete in the plane of reflection.

As the preceding results were obtained with a surface inclined 45° $23\frac{1}{2}'$ to the axis, I was anxious to observe the effects produced by the Faroë crystals, where the natural faces are nearly in the plane of the axis. I accordingly repeated the experiments with a variety of these crystals, and in every case I observed the same phenomena. In the azimuth of 90°, where the polarising angle is 58° 14′, the pencil was polarised a degree or two out of the plane of reflection. In the azimuth of 45°, where the polarising angle is about

56° 16′, the change of polarisation is about 40°; and in the azimuth of 0°, where the polarising angle is 54° 18′, the change of polarisation was a little more than 90° when the obtuse angle was farthest from the eye, and about 106° when the obtuse angle was nearest the eye. In all these positions the image reflected from the surface of the oil and the spar, is nearly as bright as that from the surface of the prism and the oil.

In order to determine the change of polarisation when the plane of reflection was perpendicular to the axis, it was necessary to have a prismatic crystal of calcareous spar with a polished summit; but I have always found this summit rough and unpolished. There was therefore no alternative but to polish an artificial face cut in this direction; and upon the application of oil of cassia, I found that in every azimuth the change of polarisation was about 75°. The colour of the image was a bright yellow, and a little blue light remained at the point of evanescence.

In extending these experiments to other crystals I have obtained similar results; but there are none so well fitted for this species of examination as calcareous spar. In applying oil of cassia to a very fine prism of chromate of lead, the direction of the polarisation was not in the slightest degree altered, as the ordinary action was not sufficiently weakened to render visible the influence of the interior force. When the plane of reflection passed through the axis of the prism, *blue* light remained in the vanishing image; but in a plane rectangular to this, the light was completely polarised, as in the experiment when the reflecting surface was in contact with air.

I now tried *rock crystal* and *oil of anise seeds*, which have nearly the same mean refraction; but on account of the great debility of the interior polarising force, it was not able to overpower or even to modify that which accompanies partial reflection. I could easily have reduced this last force still farther till it came under the dominion of the first; but the reflecting power would have been reduced in the same proportion, and would not have been capable of driving back a number of rays sufficient to form a perceptible image. But though the polarisation is not changed at the separating surface of the oil and the rock crystal, yet the character of the reflected light is modified in a very remarkable manner. When the plane of reflection from one of the sides of the prism of rock crystal was in the direction of the axis, or in 0° of azimuth, the reflected image was a *deep blue* of very little intensity; whereas in a rectangular direction, where the azimuth was 90°, it was of a brick red colour, and much more luminous. On one of the faces of the pyramid, in azimuth 0°, the tint was a *brilliant pink*, intermediate between the *red* and the *blue*; and on the same face, in 90° of azimuth, it was of a *brick red* colour as before. These variations are obviously related to the axis of double refraction, and indicate the extension of its force within the sphere of partial reflection. The origin of the colours themselves, I shall soon have occasion to explain, in a paper on the action of uncrystallized surfaces.*

3. *General results deduced from the preceding experiments.*

Had it been in my power to command a series of the most

* This Paper was read before the Royal Society of Edinburgh, on the 4th January 1819.

perfect crystals, or to communicate to artificial faces that high polish which nature often exhibits, I might have obtained a more complete generalisation of the preceding phenomena. Limited, however, as the investigation has been by these causes, it still presents us with several views of great generality and interest.

FIRST. The force of double refraction and polarisation extends without the surface of crystals, and within the sphere of the force which produces partial reflection.

SECOND. The change in the angle of polarisation produced by the interior force, depends on the inclination of the reflecting surface to the axis of the crystal, and also on the azimuthal angle which the plane of reflection forms with the principal section.

In any given surface, where A and A″ are the minimum and maximum polarising angles, viz. in the azimuth of 0° and 90°, the polarising angle A′ at any intermediate azimuth α, may be found by the formula

$$A' = A + \text{Sin.}^2\,\alpha\,(A'' - A).$$

In the rhomboidal surfaces of calcareous spar

$$A'' - A = 138'.$$

THIRD. The change in the direction of the polarisation must be produced after the ray has suffered reflection; for if the change preceded reflection, the reflecting force would have polarised it in the plane of reflection, whatever had been the direction of its previous polarisation.

FOURTH. The change in the direction of the polarisation depends upon the angle which the incident ray forms with the axis of the crystal, and takes place in such a manner that if

φ = angle of incident ray with the axis; and

C = change in the direction of the polarisation, we shall have

$$\text{Sin. } \tfrac{1}{2} C = \sqrt{\text{Sin. } \varphi}.$$

If we make

A = complement of the inclination of the reflecting plane to the axis;

α = azimuth of the plane of incidence with the principal section; and

i = angle of incidence reckoned from the perpendicular, we shall have

$$\text{Cos. } \alpha \times \text{Tang. A} = \text{Tang. } z, \text{ and}$$
$$\text{Cos. } \varphi = \frac{\text{Cos. A} \times \text{Cos. } (i \pm z)}{\text{Cos. } z}.$$

In one of the ordinary rhomboïdal surfaces where the inclination to the axis is $45^\circ\ 23\frac{1}{2}'$, A = $44^\circ\ 36\frac{1}{2}'$; and with oil of cassia i or the incidence of the mean ray, when the polarisation is complete, is about $45^\circ\ 17'$. I have assumed it at $45^\circ\ 23\frac{1}{2}'$ (which will be more correct for the mean luminous ray than $45^\circ\ 17'$) for the purpose of making the change of polarisation commence with zero in 0° of azimuth.

Upon these principles I have computed the following table, which shows the change in the direction of the polarisation, corresponding to any azimuth and any inclination of the incident ray with the axis.

Table *showing the change in the direction of the polarisation in different azimuths.*

Azimuth.	Inclination of incident ray to the axis.	Change in the direction of the polarisation.
°	° ′	° ′
0	0 0	0 0
10	6 54	40 36
20	16 50	65 6
30	23 0	77 22
40	29 24	88 52
45	32 38	94 34
50 57½′	36 29	100 54
60	42 17	110 10
70	48 32	120 0
80	54 37	129 8
90	60 0	137 0
100	65 56	145 48
110	71 4	153 4
120	75 42	159 48
129 2½′	79 28	165 3
135	81 41	169 46
140	83 22	170 34
150	86 12	174 52
160	88 18	177 36
170	89 33	179 28
180	90 0	180 0

The results in the preceding table enable us to explain the phenomenon described in p. 154. As the interposed oil of cassia has a prismatic form and a very high dispersive power, the blue and the red rays are incident at different angles with the axis, and therefore the change in the direction of their polarisation must be different. The nearest approximation to evanescence in one of the images, belongs to the mean ray of the spectrum, and therefore at this point the image that should have vanished, must consist of blue and red light, one of which will disappear *before*, and the other *after*, the mean ray.

I have the honour to be, &c. &c. &c.

DAVID BREWSTER.

To the Right Hon. Sir Joseph Banks, Bart.
G. C. B. P. R. S. &c. &c. &c.

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